Defect correlators in AdS$_2$/CFT$_1$

[2204.01659], [2106.00689], [2004.07849], [2201.04104]

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AdS$_2$/CFT$_1$

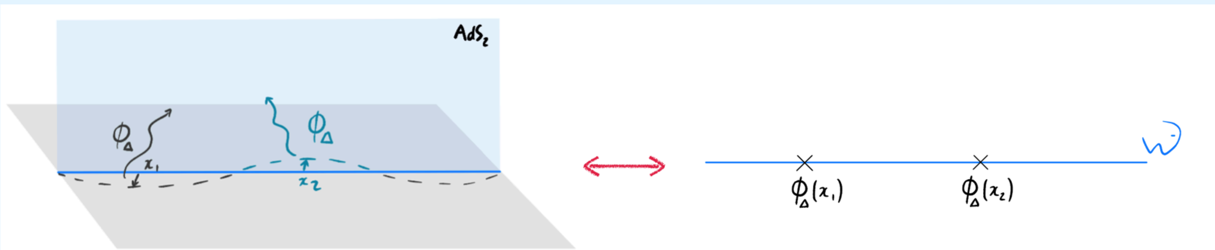
Analytic bootstrap and Witten diagrams for the ABJM Wilson line as defect CFT$_1$ [2004.07849]
***JHEP* 08 (2020) 143 [Bianchi, Bliard, Forini, Griguolo, Seminara]**

Examples of CFT$_1$ systems

- **Subsector** of higher dimensional theories ['05 Drukker Kawamoto]['17 Giombi, Roiban,Tseytlin]
- **Boundary** of QFT in AdS$_2$
- **Defect theories**

An example which includes both the case of a QFT in AdS$_2$ and a defect theory is the AdS$_2$/CFT$_1$ setup of a minimal string surface with a Wilson line at its boundary. We focus on the following dual system ['08 Aharony, Bergman, Jafferis, Maldacena]['08 Drukker, Plefka, Young] :

Duality setup



AdS$_2$ worldsheet excitations ⟷ Operator insertions on the Wilson line defect

Gauge-fixed type IIA minimal string in $AdS_4 \times \mathbb{C}P^3$ ⟷ Wilson line in 3D Chern Simons theory

$$S_B = T\int d^2\sigma \sqrt{G_{AdS_4\times \mathbb{C}P^3}} \longleftrightarrow \mathscr{W} = \mathrm{STr}\left(\mathrm{Pexp}\left(-i\oint dt \mathscr{L}(t)\right)\right)$$

Boundary fluctuations propagating in AdS$_2$ ⟷ Operators inserted on the Wilson line

$\mathbb{C}P^3$ massless fluctuations $w^a, \bar{w}_a$ ⟷ $\mathbb{O}^a, \bar{\mathbb{O}}_a$ $\Delta = 1$ Bosonic operators
Remaining AdS_4 directions $X, \bar{X}$ ⟷ $\mathbb{D}, \bar{\mathbb{D}}$ $\Delta = 2$ **Displacement Operator**

Large T limit computed perturbatively using Witten diagrams ⟷ Strong coupling limit computed perturbatively using analytic bootstrap

Superspace structure

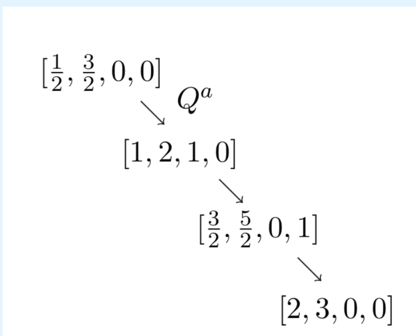
Φ has a 4-point correlator whose expansion in grassmann variables θ gives all the 4 point correlators

$$\langle \mathbb{F}\bar{\mathbb{F}}\mathbb{F}\bar{\mathbb{F}}\rangle^{(1)} = \langle \Phi\bar{\Phi}\Phi\bar{\Phi}\rangle|_{\theta=0} = \frac{1}{x_{12}x_{34}} f(z) \qquad z = \frac{x_{12}x_{34}}{x_{13}x_{24}}$$

$$\langle \mathbb{O}_{a_1}\bar{\mathbb{O}}^{a_2}\mathbb{O}_{a_3}\bar{\mathbb{O}}^{a_4}\rangle^{(1)} = (\Pi_i \partial_{\theta_i^{a_i}})\langle \Phi\bar{\Phi}\Phi\bar{\Phi}\rangle|_{\theta=0}$$

$$= \frac{1}{(x_{12}x_{34})^2}\left(\delta^{a_2}_{a_1}\delta^{a_4}_{a_3}(f(t) - tf'(t) + t^2 f''(t)) + \delta^{a_4}_{a_1}\delta^{a_2}_{a_3}(t^2 f'(t) + t^3 f''(t))\right)|_{t=z}$$

The chiral superfield



$$\Phi = \mathbb{F} + \theta^a \mathbb{O}_a + \theta^a\theta^b \epsilon_{abc}\Lambda^c + \theta^a\theta^b\theta^c\epsilon_{abc}\mathbb{D}$$

The **Witten diagram** computation and the **analytic bootstrap** agree at first order and give

$$f^{(1)}(t) = z\left(r_1(z)\log(z) + p_1(z(1-z)) + r_1(1-z)\log(1-z)\right) \qquad \textbf{(3)}$$

$$f^{(2)}(t) = z\left(t_2(z)\log(z)^2 + s_2(z)\log(z)\log(1-z) + p_2(z(1-z)) + r_2(z)\log(z) + z \leftrightarrow 1-z\right) \qquad \textbf{(*)}$$

Goals: Solve CFT$_1$ exactly.
Find information about higher dimensional embedding theories.
Learn about QFT in curved space.

Perturbative computations

Notes on n-point Witten diagrams in AdS$_2$ [2204.01659] (To appear in **JPhysA) [Bliard]**

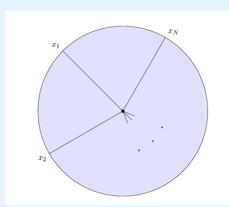
Correlation functions are computed perturbatively with **Witten diagrams** ['99 D'Hoker, Freedman, Mathur, Matusis, Rastelli]

$$G_{B\partial B}(y, x; x_i) = \left(\frac{y}{y^2 + (x - x_i)}\right)^\Delta \qquad G_{BB}(u) = u^{-1}{}_2F_1(\Delta, \Delta, 2\Delta, -\frac{2}{u}) \qquad u = \text{Geodesic distance in AdS}_2$$

Bulk to boundary propagator — Bulk-to-bulk propagator

Contact Witten diagrams correspond to the first order connected n-point correlators in a QFT in AdS$_2$ with a single ϕ^n interaction term and are computed with the integral

$$\langle \phi_\Delta(x_1)\dots\phi_\Delta(x_n)\rangle = I(x_i) = \int \frac{dxdy}{y^2}\Pi_{i=1}^n\left(\frac{y}{y^2 + (x - x_i)^2}\right)^\Delta$$

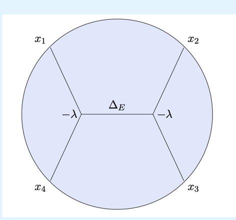


In the massless case ($\Delta = 1$), this can be computed for all n using **contour integration** for the x variable and a partial fraction decomposition for the z-variable.

$$I_{\Delta=1,n}(x_i) = \frac{\pi}{2(2i)^{n-2}}\sum_{j=1}^{n-1}\sum_{i\neq j}^{n-1}\frac{(x_i - x_j)^{n-4}}{\Pi_{k\neq j\neq i}(x_k - x_j)(x_i - x_k)}\ln(\frac{x_i - x_j}{2i})$$

Exchange Witten diagrams are related to contact diagrams by the action of the conformal Casimir and can be computed by solving the differential equation: ['99 D'Hoker, Freedman , Rastelli]

$$\left(\mathscr{C}^{(2)}_{23} - \Delta_E(\Delta_E - 1)\right)$$



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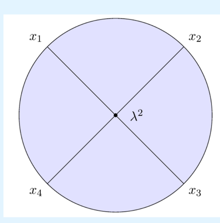

For example, the exchange diagram in the t channel gives

$$I^{\Delta=1,\Delta_E=1}_{exch} = \frac{\pi}{4(t_{13}t_{24})^2}\frac{c_1 + c_2\log(z^2) + 6\mathrm{Li}_3(z) - \mathrm{Li}_2(z)\log(z^2)}{(1-z)^2}$$

Goals: Use the n-point contact diagrams as a basis to compute and bootstrap other tree level diagrams.
Provide a tool to make AdS$_2$ computations straightforward.
Motivate the HPL ansatz and letters as the input for the analytic bootstrap.
AdS Superstring models can be studied non-perturbatively via discretised models [**Bliard**, Costa, Forini, Patella]

The analytic bootstrap

Analytic bootstrap and Witten diagrams for the ABJM Wilson line as defect CFT$_1$ [2004.07849]
***JHEP* 08 (2020) 143 [Bianchi, Bliard, Forini, Griguolo, Seminara]**
Bootstrap² of ABJM defect Wilson line -In progress- [Bliard]

Conformal symmetry in $1d$ will fix all but $n-3$ independent variables, called **conformal cross ratios**, in an n-point correlator. This fixes the 1, 2 & 3 point functions

$$\langle\phi_\Delta(x)\rangle = \delta_{\Delta,0} \qquad \langle\phi_{\Delta_1}(x_1)\phi_{\Delta_2}(x_2)\rangle = \frac{\delta_{\Delta_1,\Delta_2}}{(x_{12})^{2\Delta}} \qquad \langle\phi_{\Delta_1}(x_1)\phi_{\Delta_2}(x_2)\phi_{\Delta_3}(x_3)\rangle = \frac{c_{\Delta_1,\Delta_2,\Delta_3}}{x_{12}^{a_3}x_{13}^{a_2}x_{23}^{a_1}}$$

Where the scaling dimension Δ specifies the operator and the spectrum $\{\Delta_i\}$ and coefficients $\{c_{\Delta_1\Delta_2\Delta_3}\}$ determine the theory thanks to the **operator product expansion** (OPE) which allows one to relate these to an n-point function. For example, the 4-point function can written as

$$\langle\phi_\Delta(x_1)\phi_\Delta(x_2)\phi_\Delta(x_3)\phi_\Delta(x_4)\rangle = \frac{1}{(x_{12}x_{34})^{2\Delta}}\sum_h c^2_{\Delta,\Delta,h}\, z^h\, {}_2F_1(h, h, 2h; z)$$

Conversely, higher point functions can provide information about the theory $\{\Delta_i\}\&\{c_{\Delta_1\Delta_2\Delta_3}\}$
In practice the conformal symmetry, the OPE and any additional symmetry (reflection positivity, protected dimensions) constrain correlators further; this is used for the conformal bootstrap ['21 Fererro, Meneghelli].

The conformal bootstrap

$$\langle\mathbb{F}\bar{\mathbb{F}}\mathbb{F}\bar{\mathbb{F}}\rangle = \frac{1}{x_{12}x_{34}}f(z) \qquad \langle\mathbb{F}\bar{\mathbb{F}}\bar{\mathbb{F}}\mathbb{F}\rangle = \frac{1}{x_{12}x_{34}}h(z)$$

Crossing symmetry imposes $f(z) = f(1-z)$

Braiding symmetry relates $h(z)|_{\log^{(l)}(z)} = f(\frac{-z}{1-z})|_{\log^{(l)}(-z)}$

Transcendentality ansatz in terms of HPLs e.g. $f^{(2)}(z) = r_2(z)\mathrm{Li}_3(z) + \dots$

Lower order **CFT data** fixes the terms multiplying $\log(z)^{(l|l>1)}$ (up to mixing)

Anomalous dimension **growth** for large dimension as $\gamma_n^{(L)} \simeq n^{2L}$

Impose **boundary conditions**

$$f(z) =_{z\to 0} O(z) \qquad h(z) =_{z\to 1} O(1-z) \qquad h(z)|_{\log(1-z)} =_{z\to 1} O\left((1-z)^3\right)$$

Use of **non-perturbative data** for low dimension operators (Localisation, integral constraints) ['21, Cavaglià, Gromov, Julius, Preti]

Goals: Bootstrap 4-point function at higher order to find strong coupling spectrum.
Bootstrap correlators in Mellin formalism.
Compute 6-point functions and look for signs of integrability at strong coupling.
Bootstrap is an inherently non-perturbative approach, ansatz is the perturbative element

Mellin space

Mellin amplitudes for 1d CFT [2106.00689] ***JHEP* 10 (2021) 095 [Bianchi, Bliard, Forini, Peveri]**

In higher dimensions, Mellin space highlights some nice properties of the correlator ['11 Penedones]

- Nice **complex analytic** structure
- Links to **scattering amplitudes**
- Redundant variables for **1d**

Defining the Mellin and Anti-Mellin transform as

$$M[g](s) = \int_0^\infty dt\, t^{-1-s} f(t) \qquad \hat{M}^{-1}[M](t) = \int_{\mathscr{C}} \frac{ds}{2\pi i} t^s M(s) \qquad t = \frac{x_{12}x_{34}}{x_{14}x_{23}}$$

We can find the general expression for 4-point contact diagrams of **all integer** scaling dimension Δ

$$M^0_{\Delta_\phi}(s) = \csc(\pi s)\cot(\pi s)P_{\Delta_\phi}(s) - \csc(\pi s)\sum_{s_i=1}^{2\Delta_\phi-1}\frac{P_{\Delta_\phi}(s_i)}{s - s_i} \qquad P_{\Delta_\phi}(s) = {}_3F_2(s, 2\Delta_\phi - s, \frac{1}{2}, 1 - \Delta_\phi; 1, 1, \frac{1}{2} + \Delta_\phi; 1)$$

Scalar theories

Considering a CFT$_1$ defined as the boundary theory of the QFT in AdS$_2$, with action

$$S = g\int\frac{dydx}{y^2}\left(\frac{1}{2}\partial_\mu\phi_\Delta\partial^\mu\phi_\Delta + \frac{1}{2}\Delta(\Delta-1)\phi_\Delta^2 + D_L\phi_\Delta^4\right) \qquad D_L\phi^4 = (\partial^{2L}\phi^2)^2 + O(\partial^{4L-2})$$

The 4-point function can be computed in Mellin space, along with the spectrum and OPE coefficients

$$M^L_\Delta(s) = \sum_{k=0}^{2L}\binom{2L}{k}M^0_{\Delta+L}(s+k) \qquad \gamma^{(1)}_{L,n}(\Delta_\phi) = \frac{\Gamma(L+\Delta_\phi)^4}{\Gamma(2L+2\Delta_\phi)}\sum_{p,k,l}(-1)^l c_{k,l}F(\Delta_\phi, L, n, p, k, l)$$

In agreement with results found independantly in ['22, Knop, Mazac]

Defect theory

We Mellin transform our perturbative result for the lowest weight correlator

$$f^{(1)}(z) \to M[f^{(1)}(z)](s) \qquad M[f^{(1)}(z)](s) = 6\Gamma(s-2)\Gamma(-1-s)$$

The differential operators relating the correlators in position space become linear transformation in Mellin space. For example to obtain the Mellin amplitude of $\langle\mathbb{F}\bar{\mathbb{F}}\mathbb{O}^a\bar{\mathbb{O}}_a\rangle$

$$\langle\mathbb{F}\bar{\mathbb{F}}\mathbb{O}^a\bar{\mathbb{O}}_a\rangle = \frac{1}{t_{12}t_{34}^2}g(t) \qquad g(t) = \sum_{k,n}a_{k,n}t^k\partial_t^{(n)}f(t)$$

$$M[g](s) = \sum_{k,n}a_{k,n}(-1)^k(k-1-s)_n M[f](s+k-n)$$

If $f(t)$ has a simple expression, **all correlators** of elements of the supermultiplet will **!**

Goals: Reveal simple structure of 1d defect correlators through the Mellin transform
Find a better suited Ansatz formulation for the Bootstrap
Find a direct computable path to flat space amplitudes
Use the Mellin formalism to find signs of integrability in AdS$_2$ systems.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942