6CCP3212 Statistical Mechanics Review Problems for Thermal physics

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1) Consider the equation of state for an ideal gas

$$PV = Nk_bT \tag{1}$$

where N is the number of particles and k_b is the Boltzmann constant. Draw the following curves on a (i) $P-V$ phase diagram and a (ii) $T-V$ phase diagram.

- An isobaric (constant pressure) curve.
- An isothermal (constant temperature) curve.
- A isochoric (constant volume) curve.
- An isentropic (constant entropy) curve.

2) Find the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ for the following functions:

- $f(x,y) = \sqrt{x^2+y^2}$
- $f(x,y) = \frac{1}{x+y^2}$
- $f(x,y) = \frac{\log(x+y)}{x^2}$

3) Find the total derivative df/dt for the following functions

- $f(x,y) = x^y$, $x(t) = t^2$, $y(t) = t$.
- $f(x) = \log x^2$, $x(y,z) = y^2 + z$, $y(t) = t^{-1}$, $z(t) = t$.
- $f(x,y,t) = xyt$, $x(t) = t^2$, $y(x,t) = x + t$.

3) For the following differentials, state whether they are *exact* or *inexact*.

- $dz = xdx + ydy$.
- $dz = ydx - xdy$.

Integrate the above equations $\int dz$ from $(x,y) = (0,0)$ to $(x,y) = (1,1)$, using

- The path $x = y$.
- The path $(x,y) = (0,0)$ to $(x,y) = (0,1)$ and then $(x,y) = (0,1)$ to $(x,y) = (1,1)$.

4) What is a **closed** system? What is an **open** system? Which of the following systems are open, and which are closed.

- An refrigerator kept at $T = 273K$.
- A car engine.
- A blender.

- A hydroelectric dam.
- A thermal flask with lid on.

5) Work done *on* a thermodynamic system is given by $PdV = -đW$ (the bar in d for $đ$ is explained in the first chapter of the lecture notes, but for now you don't have to worry about it). Find the work done on the an ideal gas system $PV = Nk_bT$ to change it from (P_1, V_1) to (P_2, V_2) for

- An isobaric process $P = C$ where C is a constant.
- An isochoric process $V = C$ where C is a constant.
- A **polytropic process** is given by the following curve

$$PV^n = C \tag{2}$$

  where C is a constant and $n > 1$ is the polytropic constant. Show that the work done on the system to change it from (P_1, V_1) to (P_2, V_2) is given by

$$W = \frac{P_1V_1 - P_2V_2}{1-n} \ . \tag{3}$$

  What happens when $n = 1$? Argue that your results suggest that work is path-dependent (we will prove this in Homework 1).

- An isothermal process $T = C$ where C is a constant.

6) Consider a mole of ideal gas at $T = 400K$ with equation of state $PV = Nk_bT$ expanding reversibly and isothermally from $V = 10L$ to $V = 30L$, where L is a unit of volume. Calculate the increase in entropy S of this process.

7) A heat engine absorbs heat Q_1 reversibly from a reservoir at $T = 300K$ and expel heat Q_2 reversibly to a reservoir at $T = 200K$. This engine has an efficiency of 25% while doing $W = 125$J of work in a cycle of work.

- Calculate Q_1.
- Calculate the change in entropy of the engine system for this single cycle of work.
- Calculate the change in the entropy for the two reservoirs, and show that the *total* change of entropy for the entire reservoir-engine systems is zero for a reversible process.

8) A mole of a monoatomic ideal gas are at a temperature of $T = 300$ K. The gas expands reversibly and isothermally to twice its original volume. Calculate the work done by the gas, the heat supplied, and the change in internal energy.