Project : Solitons and Vortices

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2017 Year 3 Semester 2
Helper Set 4

1. Read the paper https://arxiv.org/pdf/1308.0605.pdf and, if you like, its more detailed companion https://arxiv.org/pdf/1308.0606.pdf. Your job is to reproduce the results of this paper, in particular, Fig 3. The primary result of the paper is that the *phase shift* $\Delta\phi$ in the collisions of two kinks, in a general periodic potential, can be analytically derived if the Lorentz factor $\gamma = (1-\beta)^{-1/2}$ of the collision is very high. Consider a kink ϕ^+ and an anti-kink ϕ^-, both with minima ϕ_0 and ϕ_1, which are traveling towards each other at velocities $\pm\beta$. In a sufficiently high velocity collision, an approximately good solution is given by simply linear superposition of the two solutions (see https://arxiv.org/abs/1005.3493 if you want to understand why)

$$\phi_{approx} = \phi^+ + \phi^- - \phi_0. \tag{1}$$

As it turns out, this approximation is almost, but not quite correct. Due to the interactions between the two kinks during the collision, the kinks will undergo a phase shift, i.e. the relative position between the two kinks will lag behind/move ahead the approximate solution ϕ_{approx}.

2. Set up your code such that the potential $V(\phi)$ is given by Eqn 22 of the paper, i.e.

$$V(\phi) = (1-\cos\phi)(1-\alpha\sin^2\phi)\ ,\ \ -1 < \alpha < 1. \tag{2}$$

Plot this potential for various values of α across the domain. How many minima are there? Choose any two neighbouring minima, ϕ_0 and ϕ_1, and construct a kink across this minima. You may use the kink solution that was provided in previous helper sets. Evolve this single kink to make sure that your code works.

3. The analytical formula for the phase shift is given by Eqn 4. Numerically integrate this (you can use Simpson's rule, or even mathematica) for different values of γ and β. Plot $\Delta\phi$ out as a function of $\gamma\times\beta$.

4. Now, set up a kink and an anti-kink (the paper uses kink-kink collisions, which you can also try if you like). Collide them at various values of $|\beta| > 0$. At sufficiently high β values, you should see the kinks "pass" through each other. Can you guess why? Find a way to measure the phase shift $\Delta\phi$ that was described in Q1.

5. Steadily increase γ – remembering that as you increase the Lorentz factor, you need to equivalently increase your resolution as the kink becomes steeper, and experiment with the collisions. Compute $\Delta\phi$ as a function of $\gamma\times\beta$, plot this out. Compare this to the analytic plot you obtained in Q3. How far can you go in terms of γ before computational resources become an issue?