Project : Solitons and Vortices

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Helper Set 2

1. Setting the speed of light $c = 1$, show that the 1+1D Klein Gordon equation

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \frac{dV(\phi)}{d\phi} = 0 \tag{1}$$

is invariant under the Lorentz transformation

$$x' = \gamma(x - \beta t)\ ,\ t' = \gamma(t - \beta x) \tag{2}$$

where the lorentz factor $\gamma = (1-\beta^2)^{-1/2}$, and the velocity $\beta \leq 1$.

2. Since the KG equation is Lorentz invariant, show that the *boosted* solution $\phi(x,t)$ is given by taking the *static* solution you found in the last helper set

$$\phi^{\pm}(x) = v \tanh\left[\pm\frac{m}{\sqrt{2}}(x - x_0)\right] \tag{3}$$

and applying the lorentz transformation Eq. (2) to it. In other words, replace x and t in Eq. (3) with their lorentz transformed coordinates, and show that it satisfies the KG equation. Vary $-1 < \beta < 1$ – what is the direction of motion?

3. Using your code, check that the boosted solution you obtained in Q2 above is indeed a solution.

4. Set up a boosted kink ϕ^+ at a location x_+, and another boosted anti-kink ϕ^- at location x_-. Choose the boosts such that the two kinks collide. Numerically solve this problem. What is the final solution after the collision?