

The p -adic Langlands Programme and Related Topics

Titles and Abstracts

May 13-17, King's College London

Laurent Berger *Formal groups and p -adic dynamical systems*

A formal group gives rise to a p -adic dynamical system. I will discuss some results about formal groups that can be proved using this point of view (rigidity, and unlikely intersections). I will also discuss the theory of p -adic dynamical systems and its relationship with (ϕ, Γ) -modules.

Christophe Breuil *Locally analytic Ext^1 and (ϕ, Γ) -modules*

Using (ϕ, Γ) -modules over the Robba ring, I will first state a conjecture on what should happen "right after" locally algebraic vectors in the locally analytic representations of $GL_n(\mathbb{Q}_p)$ associated via (patched) completed cohomology to n -dimensional de Rham representations of $\text{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ with distinct Hodge-Tate weights. I will then give several results on this conjecture. This is joint work with Yiwen Ding.

Gabriel Dospinescu *1-dimensional p -adic representations of p -adic groups*

I will present what we know (not much...) about the existence of infinitesimal characters for the locally analytic vectors in some Banach representations of p -adic groups, and how this yields nontrivial finiteness properties of these representations. I will focus most on 1-dimensional representations. This is joint work with Benjamin Schraen and Vytautas Paskunas.

Jessica Fintzen *Representations of p -adic groups*

In the 1990s Moy and Prasad revolutionized the (complex and mod ℓ) representation theory of p -adic groups by showing how to use Bruhat-Tits theory to assign invariants to representations of p -adic groups. The tools they introduced resulted in rapid advancements in both representation theory and harmonic analysis -- areas of central importance in the classical Langlands program. A crucial ingredient for many results is an explicit construction of (types for) representations of p -adic groups. In this talk I will indicate why, survey what constructions are known and present recent developments based on a refinement of Moy and Prasad's invariants.

Eugen Hellmann *The space of trianguline representations and the derived category of the Iwahori Hecke algebra*

We discuss some newer (and some older) results on the geometry of the space of trianguline representations, and their implications on the locally analytic Jacquet module of a hypothetical p -adic local Langlands correspondence. This leads to a conjecture about the derived category of the Iwahori-Hecke algebra and the derived category of coherent sheaves on a stack of L -parameters. In turn, this conjecture has implications on the computation of locally algebraic vectors in the p -adic world.

Florian Herzig *On the existence of supersingular representations*

Suppose that G is a connected reductive group over a p -adic field F . We show that the group $G(F)$ admits an irreducible admissible supersingular, or equivalently supercuspidal, representation over any field of characteristic p . So far this was only known for very few groups of low rank, like GL_2 . This is joint work with Karol Koziol and Marie-France Vigneras.

Sean Howe *p -adic automorphic forms, Igusa varieties, and functoriality*

Functions on the (big) Igusa varieties of Caraiani-Scholze furnish a rich supply of p -adic Banach spaces equipped with prime-to- p Hecke actions. These include some spaces of completed cohomology for anisotropic groups as well as spaces closely related to coherent constructions of p -adic automorphic forms, but also many new unstudied spaces. In this talk, we explain why spaces of functions on big Igusa varieties are natural p -adic analogs of classical spaces of archimedean automorphic forms, and how the geometry of perfectoid Shimura varieties can be used to compare their completed Hecke algebras. In addition to the prime-to- p Hecke action, an Igusa variety also admits a commuting action of a large p -adic group, and at the level of functions we expect this action to encode a local p -adic Langlands functoriality. This functoriality principle remains largely mysterious, but, time permitting, we will discuss some hints coming from classical modular forms.

Yongquan Hu *On the $\bmod p$ cohomology of Shimura curves*

The $\bmod p$ local Langlands correspondence is well-understood for $GL_2(\mathbb{Q}_p)$, but is still very mysterious in other cases. In this talk, I will discuss some results on the $\bmod p$ correspondence for $GL_2(F)$ when F is a finite unramified extension of \mathbb{Q}_p , in the context of the Buzzard-Diamond-Jarvis conjecture. This is joint work in progress with Haoran Wang.

Mahesh Kakde *Explicit formulae for Gross-Stark units and Hilbert's 12th problem*

In this talk I will report on my joint work in progress with Samit Dasgupta on the tower of fields conjecture first formulated by Gross. This proves a conjecture of Dasgupta on explicit p -adic analytic formulae for Gross-Stark units. These units, when considered for all primes of a totally real number field F , generate the maximal abelian CM extension of F and therefore our work can be considered as giving a p -adic analytic solution to Hilbert's 12th problem. Further, the tower of fields conjecture also proves a conjecture of Dasgupta and Spiess which gives a p -adic analytic formulae, in terms of Eisenstein cocycles, for the characteristic polynomial of the Gross regulator matrix.

Brandon Levin *The Breuil-Mezard conjecture for potentially crystalline deformation rings*

The Breuil-Mezard conjecture predicts the geometry of local deformation rings with p -adic Hodge theory condition in terms of modular representation theory. I will reformulate this conjecture in terms of the Emerton-Gee moduli stack of $\bmod p$ Galois representations. I will then describe joint work in progress with Daniel Le, Bao V. Le Hung, and Stefano Morra where we prove the conjecture in generic situations for a class of potentially crystalline deformation rings. The key ingredient is the construction of a local model for the singularities of regular weight potentially crystalline deformation rings.

Ruochuan Liu *On p -adic Riemann-Hilbert functors*

I will report recent progress on p -adic Riemann-Hilbert correspondence for p -adic étale local systems on rigid analytic varieties and some related applications.

David Loeffler *Euler systems for automorphic Galois representations*

Euler systems are compatible families of cohomology classes for global Galois representations, which play a fundamental role in proving most of the known cases of the Bloch–Kato conjecture and the Iwasawa main conjecture. I will explain a strategy for constructing Euler systems for representations appearing in the cohomology of Shimura varieties, focussing on the cases of $GSp(4)$ and $GU(2, 1)$. In particular, I will explain a surprising link with the theory of branching laws for smooth representations.

Judith Ludwig *Perfectoid Harris-Taylor Shimura varieties and a quotient of the Lubin-Tate tower*

Let K/\mathbb{Q}_p be a finite extension and consider the parabolic subgroup $P(K)$ of $GL_n(K)$ of block form $(n-1, 1)$. In this talk I will explain how to construct the quotient of the infinite-level Lubin-Tate space by the parabolic $P(K)$ as a perfectoid space. For this we prove some perfectoidness results for certain Harris-Taylor Shimura varieties at infinite level. As an application of the quotient construction we show a vanishing theorem for Scholze's candidate for the $\bmod p$ Jacquet-Langlands and the $\bmod p$ local Langlands correspondence. This is joint work with C. Johansson.

Rachel Ollivier *A derived Hecke algebra in the context of the mod p Langlands program*

Given a p -adic reductive group G and its (pro- p) Iwahori-Hecke algebra H , we are interested in the link between the category of smooth representations of G and the category of H -modules. When the field of coefficients has characteristic zero this link is well understood by work of Bernstein and Borel.

In characteristic p things are still poorly understood. In this case the role of the pro- p Iwahori-Hecke algebra H is played by a differential graded Hecke algebra. In particular, by work of Peter Schneider, the module category over the d.g. Hecke algebra is equivalent to the derived category of smooth representations of G .

Unlike in the case of H , we know little about the structure of this d.g. Hecke algebra. In this talk I will report on joint work with Peter Schneider where we take the first steps in this direction by studying the cohomology of the d.g. Hecke algebra.

Lue Pan *Fontaine–Mazur conjecture in the residually reducible case*

We prove the modularity of some two-dimensional residually reducible p -adic Galois representations over \mathbb{Q} . To do this, we generalize Emerton’s local-global compatibility result and devise a patching argument for completed homology in this setting.

Tobias Schmidt *Mod p Hecke algebras and dual equivariant cohomology*

Let H be Vignéras’ pro- p Iwahori-Hecke algebra with mod p coefficients of a p -adic GL_n . Its finite part acts on the cohomology of the mod p dual flag variety by means of classical Demazure operators. We explain how this finite representation can be uniquely extended to a natural representation of the whole algebra H and give some of its properties. Compatibility of the construction with various operations on the dual group give applications to the mod p local Langlands correspondence. If time permits, we discuss the cases of $n = 2$ and $n = 3$ in some detail. This is joint work with C. Pépin.

Liang Xiao *On the ghost conjecture of Bergdall and Pollack*

In a series of papers, Bergdall and Pollack gave a very interesting and precise conjecture on slopes, namely, the p -adic valuations of the eigenvalues for the U_p action on the space of modular forms. This, if true, would imply many long-standing conjectures on the question of slopes, notably Gouvea-Mazur’s conjecture and Gouvea’s conjecture on slope distribution. In this talk, I will report on an ongoing project with Ruochuan Liu, Nha Truong, and Bin Zhao. We first give a local reformulation of this conjecture in the framework of p -adic local Langlands for $GL_2(\mathbb{Q}_p)$. Next, we provide some theoretic evidence of this conjecture, and suggest a possible approach to our local conjecture. Finally, we will discuss some applications of this result, to the folklore conjecture of Breuil-Buzzard-Emerton on slopes of Kisin deformation spaces, and to the irreducibility of some eigencurves.

Lynnelle Ye *Slopes in eigenvarieties for definite unitary groups*

We will give bounds on the eigenvalues of the U_p Hecke operator appearing in Chenevier’s eigenvarieties for automorphic forms on definite unitary groups. These bounds generalize ones of Liu-Wan-Xiao for dimension 2, which they used to prove the Coleman-Mazur-Buzzard-Kilford conjecture in that setting, to all dimensions. We will then discuss the ideas of the proof, which uses Johansson-Newton’s extended eigenvarieties and goes through the classification of automorphic representations that are principal series at p , and a geometric consequence.

Sarah Zerbes *Euler systems and p -adic L -functions for $GSp(4)$*

I will explain the construction of a p -adic L -function attached to the spin representation of a genus 2 Siegel modular form. I will also sketch a strategy for proving an explicit reciprocity law, relating values of this p -adic L -function to the $GSp(4)$ Euler system. This is work in progress with David Loeffler, Vincent Pilloni and Chris Skinner.