P. Qi, H. Liu, L. Seneviratne, K. Althoefer, "Towards Kinematic Modeling of a Multi-DOF Tendon Driven Robotic Catheter", in Proc. 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), Aug. 2014.

Towards Kinematic Modeling of a Multi-DOF Tendon Driven Robotic Catheter

Peng Qi, Hongbin Liu, Lakmal Seneviratne, Kaspar Althoefer

***Abstract*— This paper presents a recent study on kinematic modeling of a remote-controlled active catheter. The tip steering motion of the catheter is actuated in a tendon-driven manner. Two antagonistic groups of tendon actuation realize the distal tip deflecting with two-degree-of-freedom (2-DOF) allowing it to reach a considerable large spatial workspace without catheter shaft rotation. However, when modeling such bending deformation, the sequential rotation approach is easily misapplied. We address this problem and introduce a novel and unified modeling methodology based on the concept of simultaneous rotation and the use of Rodrigues' rotation formula. An accurate model is created for robotic catheters and also can be generalized to common multi-tendon-driven continuum manipulators. It is essential in achieving accurate control and improving autonomous execution of command tracking tasks.**

I. Introduction

Minimally invasive surgery (MIS) is becoming more and more common in hospitals during the past few years, with benefits such as smaller incisions, less pain and faster recovery [1]. Cardiac catheterization is a typical procedure in this context and has been used to diagnose and treat a number of cardiac complications, such as cardiac arrhythmias or heart valve disease. During an interventional cardiac procedure, a catheter sheath is first inserted through a peripheral blood vessel, and then employing a steerable catheter tip moved up into the desired sites of the heart [2]. Overall, the procedure has two distinct phases: one is insertion stage in the blood vessels, and the other is manipulation inside the heart. In this study, we focus on the second stage and investigate the articulation of a continuum-style catheter steered by multiple tendons.

Conventional catheterization procedures require the surgeons to manually advance and steer both the sheath and the catheter – a difficult task, often only mastered well after extensive training and practice, and also potentially exposing the medical staff to high radiation dose, because of x-ray based image guidance. It is noted that inferior visualization and limited controllability result in a low positioning accuracy of the catheter tip. As a consequence, robotic positioning of the remote-controlled steerable catheter is of interest for cardiac catheter based procedures. Recent advances in both medical robotics, sensing and visualization techniques have led to an increased interest in robotic control and navigation assistance for cardiac catheterization procedures. Current commercialized products include the Sensei® X robotic catheter system (Hansen Medical) which facilitates the access of an ablation catheter to the heart with pull-wire actuation and intuitive 3D control [3]. The Niobe® ES magnetic navigation system (Stereotaxis) is another ingenious concept which integrates the catheter tip with a magnetic implant and controls its steering by changing the magnetic field produced outside the patient [4]. A detailed review and comparison of today's robotic catheterization systems is given in [5] highlighting the advancements in the field and the applicability in a clinical environment.

For both aforementioned systems, one of the main enabling components is a function-orientated steerable catheter. Compared with their conventional counterparts, steerable robotic catheters are superior in aspects of function, maneuverability, bending degrees of freedom and operation precision [6]. According to their different actuation modes, steerable catheters are usually classified into active catheter (including tendon-driven catheter, hydraulic-driven catheter and electric-actuated catheter) and magnetic catheter. Fu et al. [6] presented a comprehensive comparison and analysis of various steerable catheters. Current technological research interests are to explore the possibilities of robotic catheter design, kinematic/dynamic modeling, intraoperative sensing (tactile as well as position), optimized real-time control and motion planning/cognitive development. It is noted that an adequate understanding of the kinematics of steerable catheter is critical, while much prior work are presented in the context of general continuum robots [7]. A fundamental framework of noncontact kinematic models can be established using beam theory [8], [9] and constant-curvature approximation [10], [11]. Webster and Jones [12] presented a comprehensive review on various kinematic modeling approaches that have been frequently applied to continuum robots and also elucidated the existing similarity among their diverse forms.

Common to most tendon-driven continuum manipulators is the following: a robotic catheter articulates due to inherent compliance and conforms based on dynamic constraints on its body [7], [8], [13]. The employed antagonistic groups of tendons shape the active deflection of steerable catheter structure respectively rotating about one of two orthogonal axes. However, traditional modeling methodologies directly adopted from continuum robots no longer apply. The reason for that is that a 2-DOF steerable catheter enables robotic steering of the distal tip of the catheter without shaft rotation and robotic tip articulation in any direction is achieved by the effort of a set of driven tendons. Therefore, the modeling approach starting from planar kinematics to the spatial case of a curve rotated out of the plane is not appropriate. Previous

P. Qi, H. Liu, L. Seneviratne and K. Althoefer are with the Centre for Robotics Research, Informatics, King's College London, WC2R 2LS, UK. (e-mail: peng.qi@kcl.ac.uk)

P. Qi, H. Liu, L. Seneviratne, K. Althoefer, "Towards Kinematic Modeling of a Multi-DOF Tendon Driven Robotic Catheter", in Proc. 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), Aug. 2014.

studies mainly investigate the planar curve of a steerable catheter and less work is directly focused on the spatial case study. To bridge this gap, this paper provides an analytical method to reveal the catheter kinematics in spatial terms focusing on spatial bending configuration using the concept of simultaneous rotation and the use of Rodrigues' rotation formula.

II. Coordinate Systems And Modeling Assumptions

A. Coordinate Systems

In order to describe all positions and orientations and derive the kinematics of the robotic catheter, we will first define the following coordinate systems as shown in Fig.1.

- Reference coordinate system $\{O_r, x_r, y_r, z_r\}$ is attached to the surface of the end of the sheath where catheter tip starts to bend. XY plane coincides with this surface and origin O_r is at the center.
- Bending plane coordinate system $\{O_b, x_b, y_b, z_b\}$ is defined so that the longitudinal plane of symmetry of the bending catheter is its XZ plane. The origin O_b coincides with the origin O_r of the reference coordinate system.
- A moving frame $\{O_e, x_e, y_e, z_e\}$ is attached to the tip of steerable catheter. Its z_e axis is tangent to the longitudinal axis of the catheter and XZ plane also coincides with the catheter bending plane.

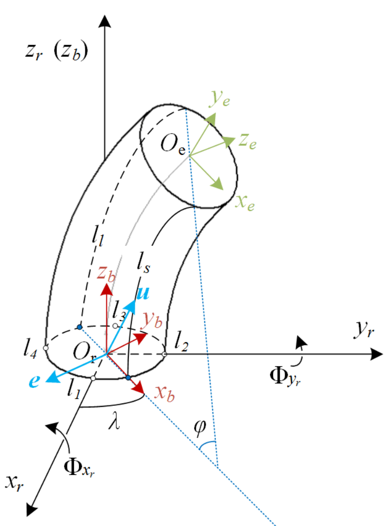

Fig. 1. Spatial bending configuration of the steerable catheter attached with coordinate systems. l_1, l_2, l_3, l_4 denote the tendon lengths and l_s, l_l denote the lengths of inner and outer boundaries of the bending plane configuration.

B. Modeling Assumptions

There arise four assumptions. One is that all the tendons are in tension and no slack during articulation. Another is that bending deformation is constant low speed motion and dynamic efforts are negligible. The third one is that the loads exerted at the tendon termination point of the distal end are uniformly distributed along the catheter. To end with, the last one is that constant-curvature assumption [12] is implemented when investigating the planar configuration in bending plane.

III. Simultaneous Rotation

The robotic catheter has 2 DOF, actuated by multiple tendons. Each antagonistic group of tendons achieves a deflection of the catheter tip in one direction. With the combined articulations of two antagonistic groups of tendons, robotic tip articulation is allowed in any direction away from the longitudinal axis of the robot. With regards to the positions and orientations of the robotic tip, we first analyze the relative rotations of the moving frame $\{O_e, x_e, y_e, z_e\}$ with respect to the reference coordinate system. We observe that the rotation axes of Φ_{x_e} and Φ_{y_e} are not fixed (Φ_{x_e} and Φ_{y_e} respectively denote the rotations of robotic tip about its local axes x_e and x_e), which complicates the analysis of the relative movements. In order to address this matter, we define a unit vector $\boldsymbol{u}(u_x, u_y, u_z)$ in the reference coordinate system to represent the spatial direction from the reference origin O_r to the origin O_e of the moving frame $\{O_e, x_e, y_e, z_e\}$. The work of describing the bending of the robotic catheter is transformed into the work of examining the rotation of the vector $\boldsymbol{u}$. In addition, a half-angle relation holds between the rotations of the distal moving frame and the corresponding rotations of the vector $\boldsymbol{u}$, i.e. we have $\Phi_{x_r} = \frac{1}{2}\Phi_{x_e}$ and $\Phi_{y_r} = \frac{1}{2}\Phi_{y_e}$, where Φ_{x_r} and Φ_{y_r} respectively, describe the rotations of the unit vector $\boldsymbol{u}$ about the axis x_r and the axis y_r of the fixed reference frame.

Considering the simultaneousness of the rotation Φ_{x_r} and the rotation Φ_{y_r}, we introduce angular velocities ω_{x_r} and ω_{y_r}, and based on the aforementioned assumption; both of them are constant finite velocities (in this case, angular velocity ω_{z_r} is equal to zero). We further represent angular velocities as vectors respectively pointing along their rotation axes:

$$\Omega_x = \begin{bmatrix} \omega_{x_r} \\ 0 \\ 0 \end{bmatrix}; \quad \Omega_y = \begin{bmatrix} 0 \\ \omega_{y_r} \\ 0 \end{bmatrix}; \quad \Omega_z = \begin{bmatrix} 0 \\ 0 \\ \omega_{z_r} \end{bmatrix} \tag{1}$$

Then, we can add angular velocities by vector addition [11]:

$$\Omega = \Omega_x + \Omega_y + \Omega_z = \begin{bmatrix} \omega_{x_r} \\ \omega_{y_r} \\ \omega_{z_r} \end{bmatrix} \tag{2}$$

It is noted that the addition operation only holds for angular velocity vectors but does not for rotations [14].

The angular velocity vector of (2) specifies a velocity quantity with a unit vector $\boldsymbol{e}$ (passing through the origin O_r of the reference coordinate system) indicating the direction of an axis of rotation and the scalar ω indicating the angular velocity. Therefore, we can also write

$$\Omega = \omega \cdot \boldsymbol{e} \tag{3}$$

which conforms to the right-hand rule.

As we have assumed the angular velocity to be constant, multiplying an observation interval T to the angular velocity ω, we can derive that the rotation action of the unit vector $\boldsymbol{u}$

about the rotation axis $\boldsymbol{e}$ is equal to ωT – hence,we realize the concept of simultaneous rotation. AN alternative derivation is also achieved via infinitesimally small rotations [15].

$$\begin{aligned}\boldsymbol{e} &= \frac{1}{\sqrt{{\omega_{x_r}}^2+{\omega_{y_r}}^2+{\omega_{z_r}}^2}}\begin{bmatrix}\omega_{x_r}\\ \omega_{y_r}\\ \omega_{z_r}\end{bmatrix}\\ &= \frac{1}{T\sqrt{{\omega_{x_r}}^2+{\omega_{y_r}}^2+{\omega_{z_r}}^2}}\begin{bmatrix}\omega_{x_r}T\\ \omega_{y_r}T\\ \omega_{z_r}T\end{bmatrix}\\ &= \frac{1}{\sqrt{{\Phi_{x_r}}^2+{\Phi_{y_r}}^2+{\Phi_{z_r}}^2}}\begin{bmatrix}\Phi_{x_r}\\ \Phi_{y_r}\\ \Phi_{z_r}\end{bmatrix}\end{aligned} \tag{4}$$

$$\omega T = T\sqrt{{\omega_{x_r}}^2+{\omega_{y_r}}^2+{\omega_{z_r}}^2} = \sqrt{{\Phi_{x_r}}^2+{\Phi_{y_r}}^2+{\Phi_{z_r}}^2} \tag{5}$$

IV. Catheter Kinematics

Kinematic modeling of the catheter deals with the derivation of a mapping from actuation lengths of the tendons via configuration space variables to task space positions and orientations of the tip. There are two sub-mappings [11]: the robot-independent mapping and the robot-specific mapping.

A. Robot-independent Mapping

The robot-independent mapping relates catheter configuration parameters (length, curvature, etc.) to task space positions and orientations of the tip. Previously, the first step of the modeling algorithm for a continuum manipulator is to describe the movement of a planar curve, and then view the curve rotated out of the plane. In the following, we will directly analyze the spatial bending configuration of the catheter based on the concept of simultaneous rotation and the use of Rodrigues' rotation formula. Continued with the preceding derivation on simultaneous rotation and given the axis $\boldsymbol{e}$ and the rotation angle ωT, we can rotate the unite vector $\boldsymbol{u}$ via Rodrigues' rotation formula [16], [17]

$$\boldsymbol{R} = \cos\theta\boldsymbol{I} + \sin\theta[\boldsymbol{s}\times] + (1-\cos\theta)\boldsymbol{s}\boldsymbol{s}^T \tag{6}$$

where $\boldsymbol{I}$ is a 3×3 identity matrix; θ specifies the rotation angle which is equal to ωT; $\boldsymbol{s}$ denotes the axis about which the vector is rotating.

The Rodrigues' rotation formula constructs a rotation matrix $\boldsymbol{R}$ to rotate a vector with the rotation $\theta(-\pi<\theta<\pi)$ about the axis $\boldsymbol{s}$.

$$[\boldsymbol{s}\times] = \begin{bmatrix}0 & -s_z & s_y\\ s_z & 0 & -s_x\\ -s_y & s_x & 0\end{bmatrix} \tag{7}$$

Multiplying the rotation matrix $\boldsymbol{R}$ on the left of unit vector $(0,0,1)^T$ which indicates the initial position of the direction vector $\boldsymbol{u}$, the resulting orientation is given by

$$\boldsymbol{u} = \boldsymbol{R}\begin{bmatrix}0\\0\\1\end{bmatrix} \tag{8}$$

With the constant-curvature assumption and reference to the bending plane in Fig. 2, we can calculate the distance D from the origin O_r to the origin O_e of the tip moving frame as

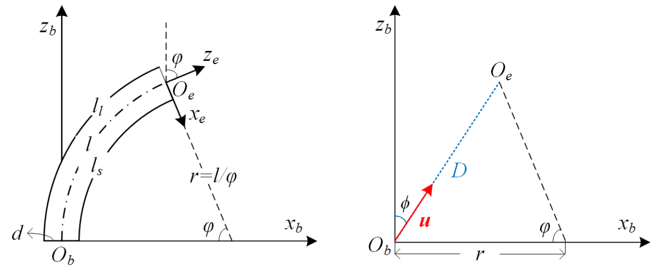

Fig. 2. Bending plane configuration of the catheter model.

$$D = 2r\sin\left(\frac{\varphi}{2}\right) \tag{9}$$

where φ is the bending angle of the curve; r is the radius and equals l/φ (l denotes the curve length of the catheter central line). Besides, we note the following half-angle relation, i.e. $\phi = \varphi/2$, where ϕ is the rotation angle of unit vector $\boldsymbol{u}$ in the bending plane.

Multiplying the length D with the rotated direction vector $\boldsymbol{u}$ which is derived from (8), we can write

$$\boldsymbol{u}' = D\boldsymbol{u} = \begin{bmatrix}{u_x}'\\ {u_y}'\\ {u_z}'\end{bmatrix} \tag{10}$$

where ${u_x}', {u_y}', {u_z}'$ are the coordinates of the position in the reference frame.

Similar with (4), the orientation of the moving frame $\{O_e, x_e, y_e, z_e\}$ can also be calculated and described in direction vector as

$$\boldsymbol{c} = \frac{2}{\sqrt{{u_x}'^2+{u_y}'^2+{u_z}'^2}}\begin{bmatrix}{u_x}'\\ {u_y}'\\ {u_z}'\end{bmatrix} \tag{11}$$

Note that the result doubles the angle of the orientation of the rotated direction vector $\boldsymbol{u}'$ as mentioned above.

B. Robot-specific Mapping

The robot-specific mapping will vary on a case-by-case basis and the relation between configuration and actuation should be a one-to-one correspondence. For our catheter case, the shape is steered by four continuously bending tendons, which is one type of frequently applied actuation strategies for continuum robots. In this tendon-driven design, the loads are exerted to the top plane via two antagonistic groups of inextensible tendons. The mapping from lengths of the tendons (l_1, l_2, l_3, l_4 as shown in Fig. 1) to catheter configuration parameters can be directly derived based on the concept and the method in [10], [12] and it can be shown that

$$\lambda = \tan^{-1}\left(\frac{l_4-l_1}{l_3-l_1}\right) \tag{12}$$

$$k = \frac{(l_1-3l_2+l_3+l_4)\sqrt{(l_4-l_2)^2+(l_3-l_1)^2}}{d\,(l_1+l_2+l_3+l_4)\,(l_4-l_2)} \tag{13}$$

where k represents the curvature of the catheter bending and holds the relations $k = 1/r$ and $\varphi = kl$. d is the distance from the centre of the catheter cross-section to the centre of the tendon.

Thus, referring back to (9), the distance between the robotic tip and the origin of the reference coordinate system can be derived, being represented by the value of parameter k. Further, combing with the rotation angle of the bending plane, we will solve another two angular components Φ_{x_r} and Φ_{y_r} that are about axes x_r and y_r, respectively.

We have now found the positions and orientations of the tip as functions of the input variables (i.e. tendon lengths l_1, l_2, l_3, l_4) and the known length l of the steerable catheter, thus completing the kinematic model.

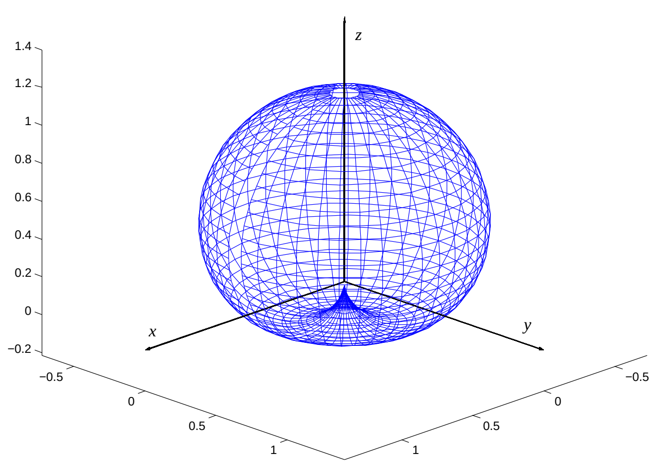

Fig. 3. Theoretical workspace of 2-DOF robotic catheter tip simulated using MATLAB®.

V. CONCLUSION

The kinematics of a robotic catheter was derived via the concept of simultaneous rotation and the use of Rodrigues' rotation formula. Our approach overcomes the lack of proper models for this type of robotic catheters that allows the articulation by means of multiple tendons whilst avoiding the rotation of the shaft. We also simulated the tip workspace of the robotic catheter using MATLAB®. The antagonistic groups of tendons are simultaneously actuated such that the robotic tip reaches all the possible sites. The entire theoretical workspace of the robotic tip is a closure surface of an inverted "apple", as shown in Fig. 3. The model is novel and unified and can apply to more general single-section cases in the continuum robot domain. It is essential in achieving accurate control and improving the execution of robot tracking tasks.

ACKNOWLEDGEMENT

The work described in this paper was partially funded by European Commission's Seventh Framework Programme under grant agreement 287728 in the framework of EU project STIFF-FLOP and the China Scholarship Council.

REFERENCES

[1] D. B. Camarillo, T. M. Krummel and J. K. Salisbury, "Robotic Technology in Surgery: Past, Present, and Future", *American Journal of Surgery*, vol. 188, no. 4A, pp. 2-15, 2004.

[2] W. Saliba, et al, "Atrial Fibrillation Ablation Using a Robotic Catheter Remote Control System Initial Human Experience and Long-Term Follow-Up Results", *Journal of the American College of Cardiology*, vol. 51, no. 25, pp. 2407-2411, 2008.

[3] Sensei® X robotic catheter system developed by Hansen Medical, Internet: http://www.hansenmedical.com/

[4] Niobe® ES magnetic navigation system developed by Stereotaxis, Internet: www.stereotaxis.com/

[5] K. J. Chun, B. Schmidt, et al, "Catheter ablation - new developments in robotics", *Herz Kardiovaskulre Erkrankungen*, vol. 30, no. 8, pp. 586-589, 2008.

[6] Y. Fu, H. Liu, W. Huang, S. Wang, and Z. Liang, "Steerable catheters in minimally invasive vascular surgery", *The International Journal of Medical Robotics and Computer Assisted Surgery*, vol. 5, no. 4, pp. 381-391, 2009.

[7] G. Robinson, J. B. C. Davies, "Continuum robots-a state of the art", *in Proceedings of IEEE International Conference on Robotics and Automation*, vol. 4, pp. 2849-2854, 1999.

[8] D. B. Camarillo, C. F. Milne, C. R. Carlson, M. R. Zinn, and J. K. Salisbury, "Mechanics modeling of tendon-driven continuum manipulators", *IEEE Transactions on Robotics*, vol. 24, no. 6, pp. 1263-1273, 2008.

[9] D. Trivedi, A. Lotfi, and C. D. Rahn, "Geometrically exact models for soft robotic manipulators", *IEEE Transactions on Robotics*, vol. 24, no. 4, pp. 773-780, 2008.

[10] M. W. Hannan and I. D. Walker, "Kinematics and the Implementation of an Elephant's Trunk Manipulator and Other Continuum Style Robots", *Journal of Robotic Systems*, vol. 20, no. 2, pp. 45-63, 2003.

[11] B. A. Jones and I. D. Walker, "Kinematics for Multisection Continuum Robots", *IEEE Transactions on Robotics*, vol. 22, no. 1, pp. 43-55, 2006.

[12] R. J. Webster, B. A. Jones, "Design and Kinematic Modeling of Constant Curvature Continuum Robots: A Review", *The International Journal of Robotics Research*, vol. 29, no. 13, pp. 1661-1683, 2010.

[13] D. B. Camarillo, C. R. Carlson, and J. K. Salisbury, "Configuration Tracking for Continuum Manipulators with Coupled Tendon Drive", *IEEE Transactions on Robotics*, vol. 25, no. 4, pp.798-808, 2009.

[14] M. F. Beatty, *Principles of Engineering Mechanics, Volume 1*. Plenum Press: New York, NY, USA, 1986.

[15] S. Stančin, and S. Tomažič, "Angle estimation of simultaneous orthogonal rotations from 3D gyroscope measurements", *Sensors*, vol. 11, no. 9, pp. 8536-8549, 2011.

[16] L. W. Tsai, *Robot analysis: the mechanics of serial and parallel manipulators*. John Wiley & Sons, 1999.

[17] J. S. Dai, *Screw Algebra and Lie Groups and Lie Algebras*, Higher Education Press, Beijing, also *Screw Algebra and Kinematic Approaches for Mechanisms and Robotics*, Springer, London, 2014.