# Large Independent Sets in LoS Networks

Joint work with **Pavan Sangha**, and **Prudence Wong**

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# Outline

## Preliminaries

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The Maximum Independent Set Problem

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Known Results

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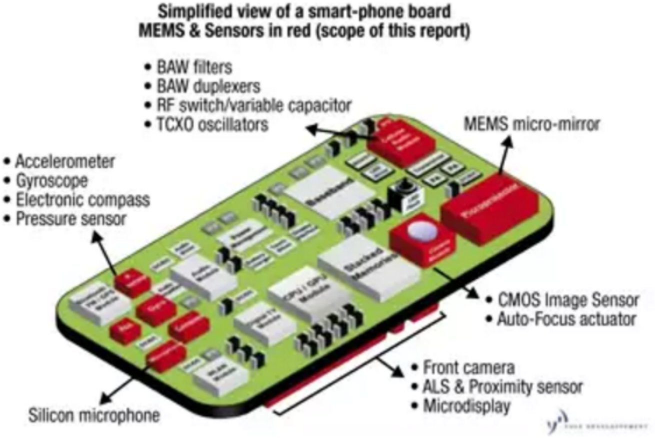
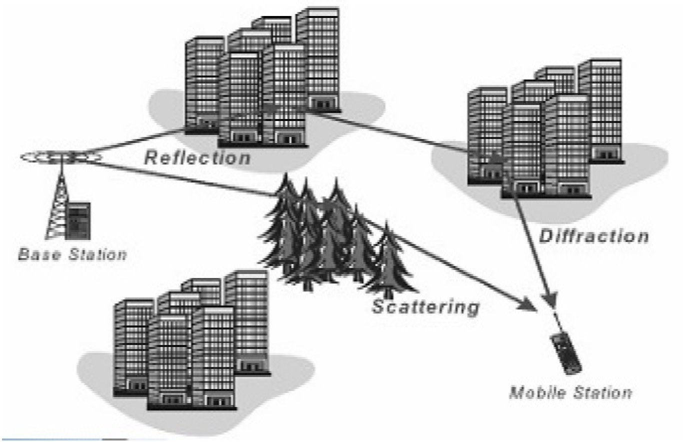
New Results

# Mobile Communication



# Mobile Communication (Issues)



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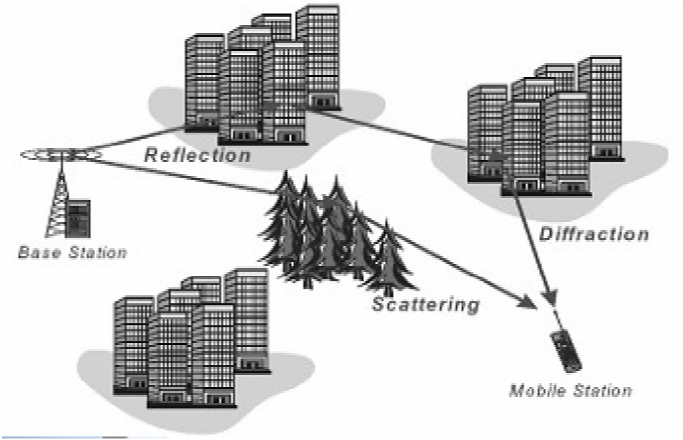
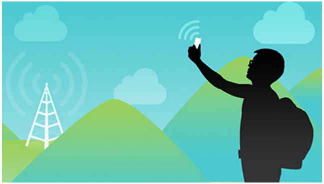
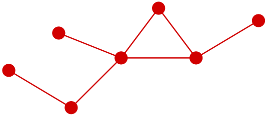


# Line of Sight Networks

## (Frieze, Klienberg, Ravi, Debany, circa 2004)



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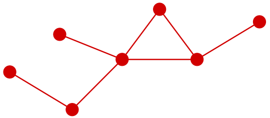

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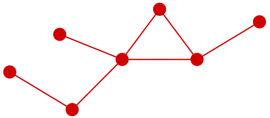
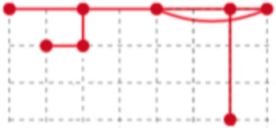
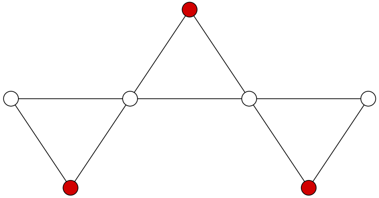




A graph $G = (V, E, w)$ is a *(narrow) Line of Sight (LoS) network* (with parameters $n$, $k$ and $\omega$) if there exists an embedding $f_G : V \to \mathbb{Z}_n^d$ (resp. with $f_G(V) \subseteq \mathbb{Z}_{n,k}^d$) such that $\{u, v\} \in E$ if and only if $f_G(u)$ and $f_G(v)$ share a line of sight and the (Manhattan) distance between them is less than $\omega$.

$\omega$ is the *range parameter* of the network.

# Independent Sets



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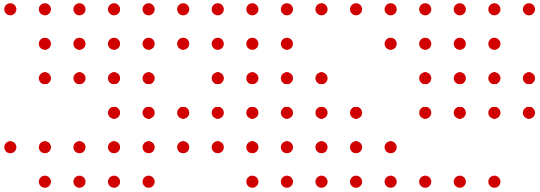
New York has many more streets than avenues.



(here junctions represented without road connections)

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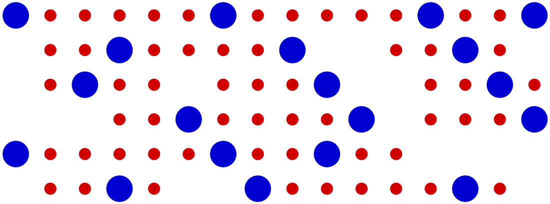

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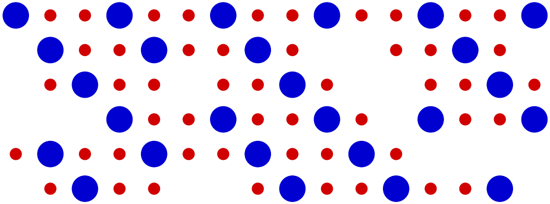

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- Hard to approximate in general

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  For fixed $\omega \geq 3$ the problem is NP-hard, there exists a natural 2-approximation algorithm, and there exists a PTAS.
- In dimension $d > 2$ the problem is also APX-hard when $\omega \geq n$
  For fixed $\omega \geq 3$ same as above but $d$-approximation algorithm

# New Results

- A maximum independent set of a (weighted) $k$-narrow $d$-dimensional LoS network with range parameter $\omega$ can be found in time $O(n(k^{(d-1)/\omega}\ \omega)^{k^{d-1}})$.

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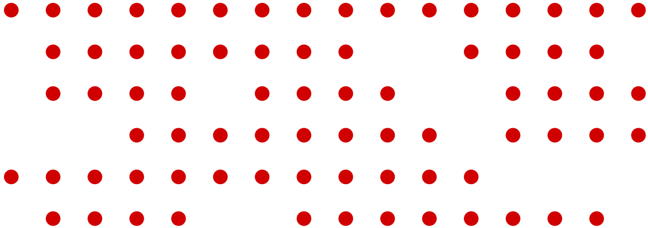
  (improves existing $O(n^2(\omega/\epsilon)^{2^{\frac{\omega+1}{\epsilon}}})$ algorithm ... which also works for $d > 2$).

# Dynamic Programming Basic Idea
## Key Observation

A narrow LoS network is uniquely described by a $k \times n$ array of zeroes and ones, encoding the vertex positions in the grid



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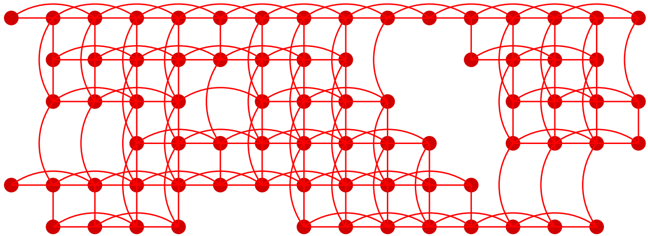


$\omega = 3$.

# Dynamic Programming

- We use a table MIS with $n$ rows and one column for each $k \times \omega$ array $W$ describing a LoS network with no edge.
- $\mathrm{MIS}(j, W)$ contains the size of the largest independent set $I$ in the first $j$ columns of $G$, such that the $\omega$ right-most columns of $I$ coincide with $W$ ($\mathrm{MIS}(j, W) = 0$ if $W$ is not a subgraph of the $\omega$ rightmost columns of $G$).

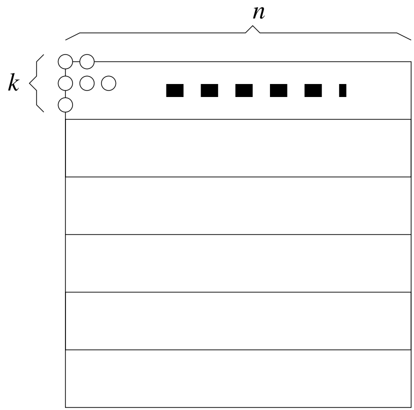
**Claim**

$\mathrm{MIS}(j, W)$ can be computed using only elements of the form $\mathrm{MIS}(j - 1, W')$ such that $W \sim W'$.

## Semi-online Algorithms

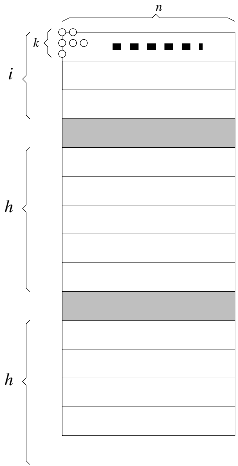
- Let $G_r$ denote the first $r\omega$ columns of $G$.
- Compute a max size independent set $I_r$ in $G_r$.
- Let $r^*$ be the smallest integer such that $|I_{r^*+1}| < (1+\epsilon)\,|I_{r^*}|$.
- To obtain a $(1+\epsilon)$-approximation, once we reach $r^*$, we remove $G_{r^*+1}$ from the graph $G$ and apply the procedure iteratively.

# Approximation Algorithm




Use $k = \omega - 1$.
Pick the largest between the even and the odd strips.

# New Approximation Scheme




Miss one strip every $h$.
One choice guarantees we get at least $h/(1+h)$ of the nodes in an optimal independent set.