

# Streaming and property testing algorithms for string processing

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Based on joint work with:

R. Clifford, P. Gawrychowski, A. Fontaine, E. Porat, B. Sach

- ▶ Pattern matching has been studied for **40+ years**
- ▶ **More than 85 algorithms**
- ▶ KMP algorithm uses  $O(|P|)$  space and  $O(|T|)$  time, and Aho-Corasick achieves similar bounds for dictionary matching
- ▶ **We can't do better:** we must store a description of the pattern(s) and we must read the whole text

**GAME OVER**

# Intrusion Detection Systems



- ▶ Large number of patterns
- ▶ Search patterns represent portions of known attack patterns and have length 1 – 30
- ▶ If only cache memory is used, the algorithm can benefit most from a high performance cache

# Outline of today's talk

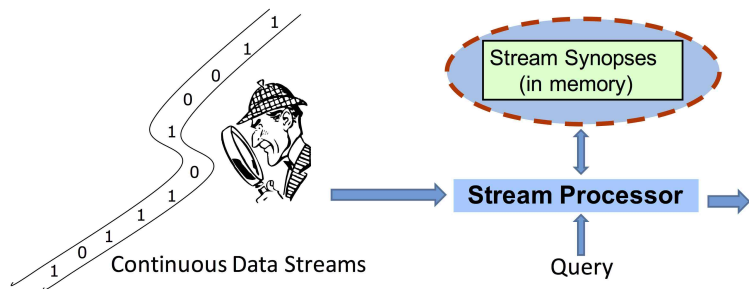
## **Streaming model**

- ▶ Exact pattern matching
- ▶ Approximate pattern matching (Hamming distance)
- ▶ Approximate pattern matching (edit distance)
- ▶ Preprocessing

## **Property testing model**

- ▶ Exact pattern matching

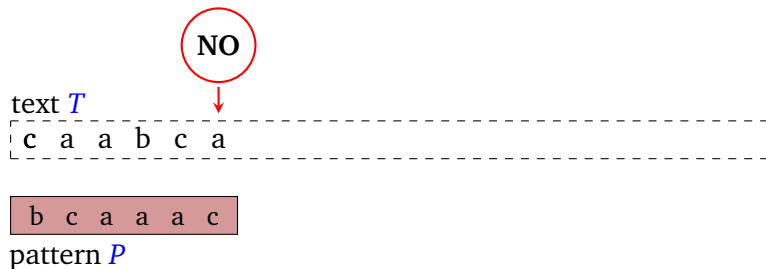
# Streaming model



We want to process the stream **on-the-fly & in small space**

# Part I: Exact pattern matching

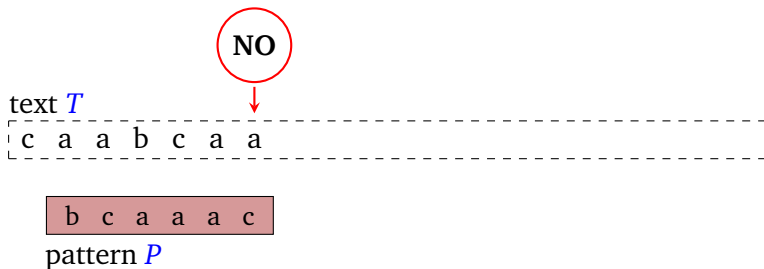
# Exact pattern matching



- ▶ **Query** = “Is there an occurrence of  $P$ ?”
- ▶ **Space** = total space used by the stream processor
- ▶ **Time** = time per position of  $T$

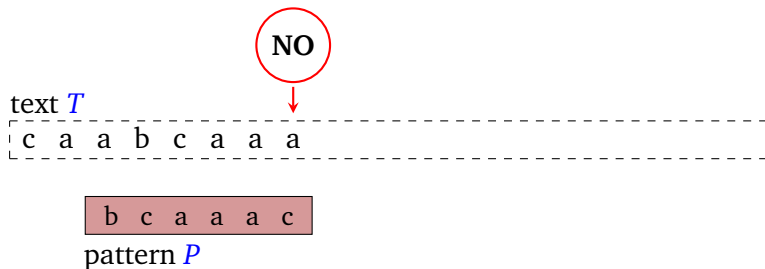


# Exact pattern matching



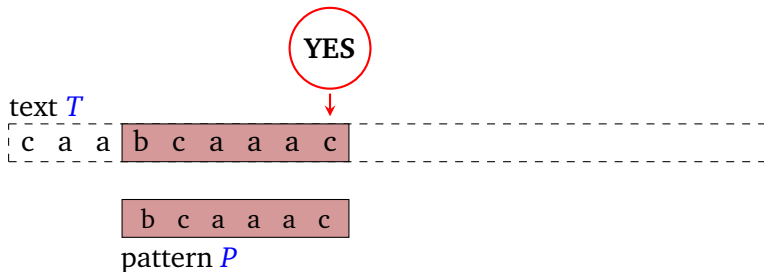
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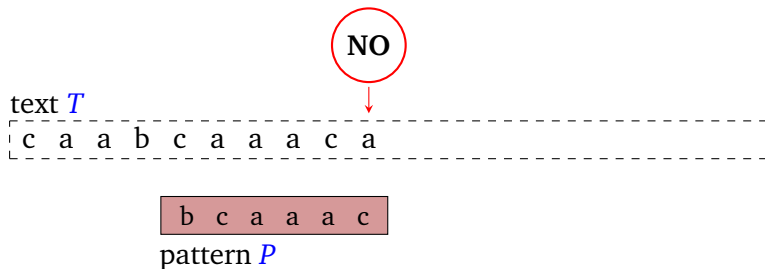
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# Karp-Rabin algorithm

## Karp-Rabin fingerprint

$$\varphi(s_1s_2 \dots s_m) = \sum_{i=1}^m s_i r^{m-i} \bmod p$$

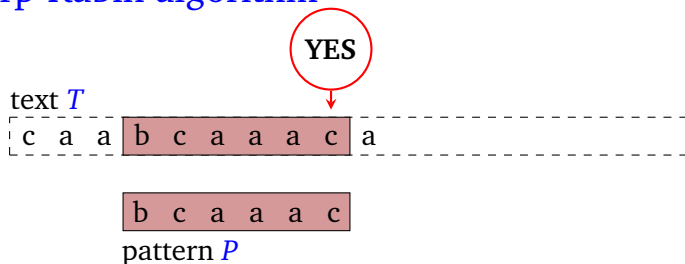
where  $p$  is a prime and  $r$  is a random integer  $\in [0, p - 1]$

## It's a good hash function

$S_1, S_2$  are two strings of length  $m$ , the prime  $p$  is large

1.  $S_1 = S_2 \Rightarrow \varphi(S_1) = \varphi(S_2)$
2.  $S_1 \neq S_2$ , lengths of  $S_1, S_2$  are equal  $\Rightarrow \varphi(S_1) \neq \varphi(S_2)$  w.h.p.

# Karp-Rabin algorithm



When a new character  $t_i = a$  arrives:

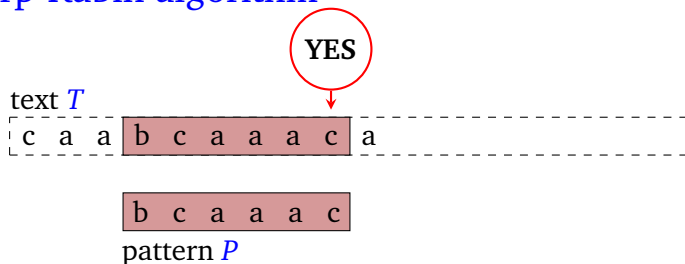
1. Compute the fingerprint  $\varphi(t_{i-m+1} \dots t_{i-1} t_i)$  in  $O(1)$  time

$$\varphi(\underline{caaacc}) = ((\varphi(\underline{bcaaac}) - br^{m-1}) \cdot r + a \bmod p)$$

2. If  $\varphi(t_{i-m+1} \dots t_{i-1} t_i) = \varphi(P)$ , output “YES”

We need  $t_{i-m}$  to update the fingerprint  $\Rightarrow$  **we must store**  $t_{i-m}, \dots, t_{i-1}$

# Karp-Rabin algorithm



K.-R. algorithm is a **streaming pattern matching algorithm** that uses  $\Theta(m)$  space and  $O(1)$  time per character of  $T$

It finds all occurrences of  $P$  in  $T$  correctly w.h.p.

# Exact pattern matching

Authors	Space <sup>1</sup>	Time
<b>Single pattern</b>		
Karp & Rabin, 1987	$\Theta(m)$	$O(1)$
Porat & Porat, 2009	$O(\log m)$	$O(\log m)$
Breslauer & Galil, 2011	$O(\log m)$	$O(1)$

<b>Dictionary of <math>d</math> patterns</b>		
Clifford, Fontaine, Porat Sach, S., 2015	$O(d \log m)$	$O(\log \log(m + d))$
Golan & Porat, 2017	$O(d \log m)$ $O( \Sigma ^\epsilon d \log(m/\epsilon))$	$O(\log \log  \Sigma )$ $O(1/\epsilon)$

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<sup>1</sup>In words



# Exact pattern matching

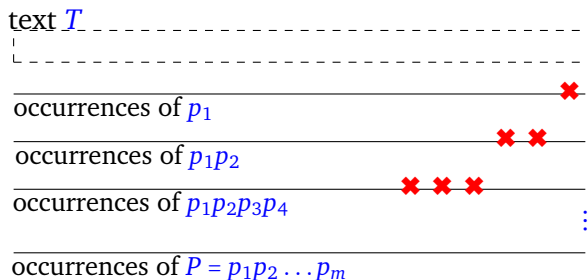
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<sup>1</sup>In words

## Porat & Porat, 2009 ★



**for** each character  $t_i$  **do**

**if**  $t_i = p_1$  **then** push  $i$  to level 0

**for** each  $j = 0, \dots, \log m - 1$

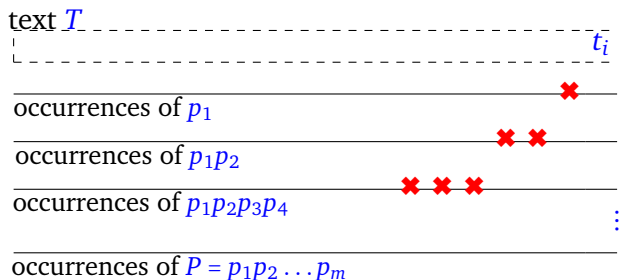
$lp \leftarrow$  leftmost position in level  $j$

**if**  $i - lp + 1 = 2^{j+1}$  **then**

      Pop  $lp$  from level  $j$

**if**  $\varphi(t_{lp} \dots t_i) = \varphi(p_1 \dots p_{2^{j+1}})$  **then** push  $lp$  to level  $j + 1$

## Porat & Porat, 2009 ★



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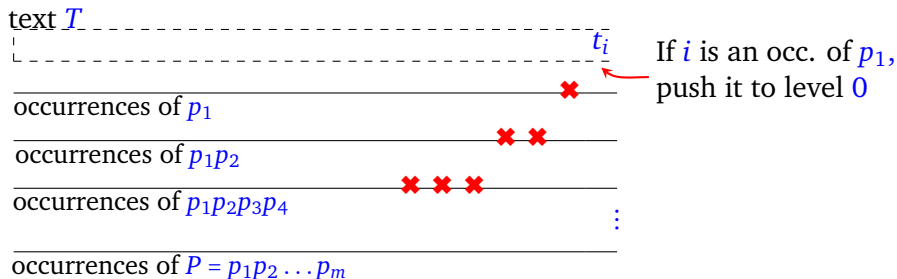
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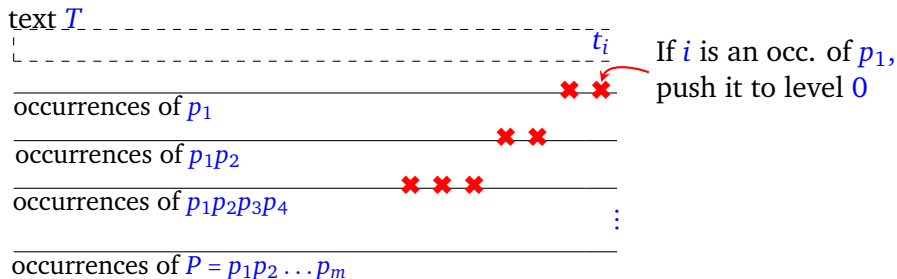
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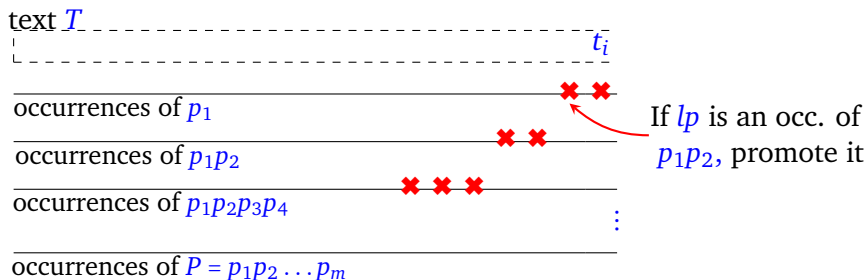
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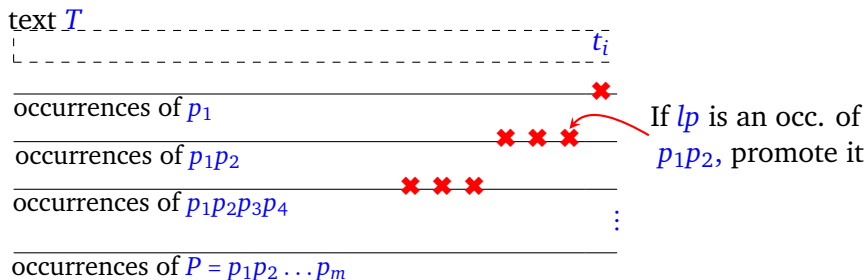
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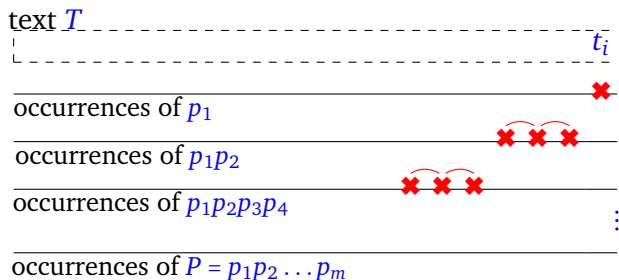
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## Porat & Porat, 2009 ★



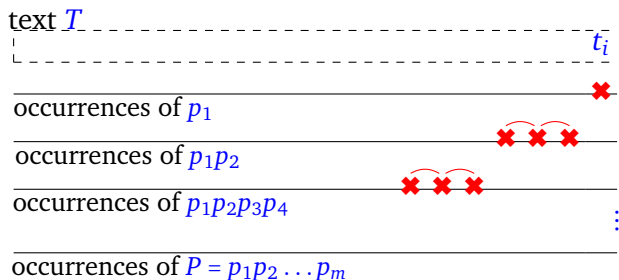
Lemma If there are  $\geq 3$  occurrences of a  $2^j$ -length string in a  $2^{j+1}$ -length string, the occurrences form a run

For each level we store:

- ▶ The leftmost and the second leftmost positions  $lp, lp'$
- ▶ The fingerprints of  $t_1t_2 \dots t_{lp}, t_{lp+1} \dots t_{lp'}$ , and  $t_1 \dots t_i$



## Porat & Porat, 2009 ★



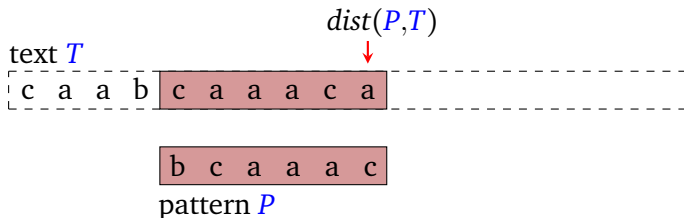
For each level we need:

- ▶  $O(1)$  space
- ▶  $O(1)$  time for updating and extracting  $\varphi(t_{lp} \dots t_i)$

Theorem Porat & Porat algorithm is a streaming pattern matching algorithm that uses  $O(\log m)$  space and  $O(\log m)$  time per character

## Part II: Approximate pattern matching

# Approximate pattern matching



- ▶ **Query** = “Distance between  $P$  and  $T$ ”
- ▶ **Distance**: Hamming, edit, ...

## Approximate pattern matching (Hamming distance)

Any streaming algorithm for computing **exact** Hamming distances must use  $\Omega(m)$  space

By **Yao's minimax principle** it suffices to consider deterministic algorithms on “hard” distribution of the inputs

text  $T$

1 0 1 1 0 0 | 0 0 0 0 0 0

0 0 0 0 0 0

pattern  $P$

$T[1, m]$  is random

After reading  $T[m]$ , the algorithm cannot go back and read one of the letters  $T[1], T[2], \dots, T[m]$ , but can restore  $T[1, m]$

Therefore, it stores a full description of  $T[1, m] \Rightarrow \Omega(m)$  space by information-theoretic ideas

## Approximate pattern matching (Hamming distance)

Any streaming algorithm for computing **exact** Hamming distances must use  $\Omega(m)$  space

By **Yao's minimax principle** it suffices to consider deterministic algorithms on “hard” distribution of the inputs

$$\text{dist}(P, T) = 3$$

text  $T$

1 0 1 1 0 0 | 0 0 0 0 0 0

0 0 0 0 0 0

pattern  $P$

$T[1, m]$  is random

After reading  $T[m]$ , the algorithm cannot go back and read one of the letters  $T[1], T[2], \dots, T[m]$ , but can restore  $T[1, m]$

Therefore, it stores a full description of  $T[1, m] \Rightarrow \Omega(m)$  space by information-theoretic ideas

## Approximate pattern matching (Hamming distance)

Any streaming algorithm for computing **exact** Hamming distances must use  $\Omega(m)$  space

By **Yao's minimax principle** it suffices to consider deterministic algorithms on “hard” distribution of the inputs

$$\text{dist}(P, T) = 2, T[1] = 3 - 2$$

text  $T$

1	0	1	1	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---

0	0	0	0	0	0
---	---	---	---	---	---

pattern  $P$

$T[1, m]$  is random

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## Approximate pattern matching (Hamming distance)

Any streaming algorithm for computing **exact** Hamming distances must use  $\Omega(m)$  space

By **Yao's minimax principle** it suffices to consider deterministic algorithms on “hard” distribution of the inputs

$$\text{dist}(P, T) = 2, T[2] = 2 - 2$$

text  $T$

1 0 1 1 0 0 | 0 0 0 0 0 0

0 0 0 0 0 0

pattern  $P$

$T[1, m]$  is random

After reading  $T[m]$ , the algorithm cannot go back and read one of the letters  $T[1], T[2], \dots, T[m]$ , but can restore  $T[1, m]$

Therefore, it stores a full description of  $T[1, m] \Rightarrow \Omega(m)$  space by information-theoretic ideas

# Approximate pattern matching (Hamming distance)

Authors	Space <sup>2</sup>	Time
<b>Single pattern, only distances <math>\leq k</math></b>		
Porat & Porat, 2009	$\tilde{O}(k^3)$	$\tilde{O}(k^2)$
Clifford, Fontaine, Porat, Sach, S., 2016	$\tilde{O}(k^2)$	$\tilde{O}(\sqrt{k})$
Clifford, Kociumaka, Porat, 2018	$O(k \log \frac{m}{k})$	$O(k \log^3 m \log \frac{m}{k})$
<b>Single pattern, <math>(1 + \varepsilon)</math>-approx.</b>		
Clifford, S., 2016	$O(\varepsilon^{-5} \sqrt{m} \log^4 m)$	$O(\varepsilon^{-4} \log^3 m)$

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<sup>2</sup>In words



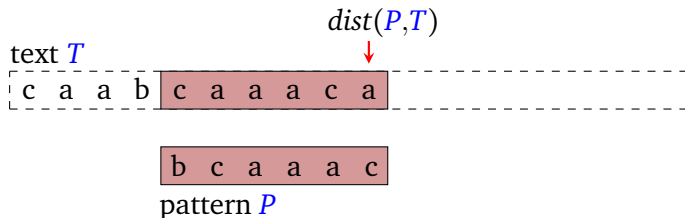
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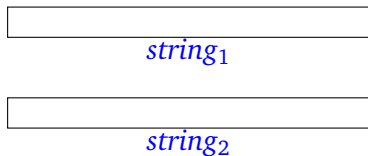
<sup>2</sup>In words

## Porat & Porat, 2009 ★



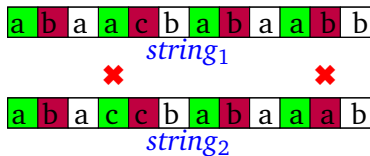
- ▶ If  $HAM(P, T) > k$ , output “NO”
- ▶ Otherwise, output  $HAM(P, T)$

## From 1 mismatch to exact pattern matching



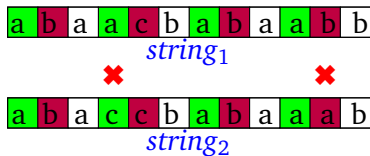
- ▶ Is  $\text{HAM}(string_1, string_2) = 1$ ?

## From 1 mismatch to exact pattern matching



- ▶ Is  $HAM(string_1, string_2) = 1$ ?
- ▶ Partition the strings into substrings of  $q$  colors
- ▶ One mismatch  $\Rightarrow$  one pair of substrings does not match
- ▶ **Hope:** If there are  $\geq 2$  mismatches, they will end up in substrings of different colors  $\Rightarrow$  at least 2 pairs of substrings do not match

## From 1 mismatch to exact pattern matching



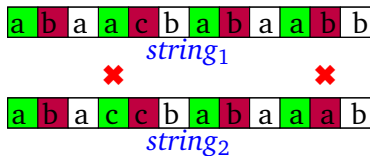
For each prime  $q \in [\log m, \log^2 m]$ :

Partition *string*<sub>1</sub> into  $q$  equi-spaced substrings

Partition *string*<sub>2</sub> into  $q$  equi-spaced substrings

**In total:**  $O(\log m)$  primes, and for each prime there are  $O(\log^2 m)$  pairs of substrings

## From 1 mismatch to exact pattern matching



Lemma There are  $\geq 2$  mismatches  $x_1, x_2 \Rightarrow$  there exists a prime  $q$  such that at least two pairs of substrings do not match

- ▶  $x_1, x_2$  in the same pair  $\Leftrightarrow x_1 - x_2 = 0 \pmod{q}$
- ▶  $m \geq x_1 - x_2$  cannot be a multiple of  $\log m$  distinct primes

## From 1 mismatch to exact pattern matching

text  $T$



pattern  $P$

Is  $\text{HAM}(P, T) = 1$ ?

**for** each position of the text  $T$  **do**

**for** each prime  $q$  in  $[\log m, \log^2 m]$  **do**

$h \leftarrow$  number of (substream, subpattern) that mismatch

**if**  $h = 0$  **OR**  $h > 1$  **return** "NO"

**return** "YES"

# From 1 mismatch to exact pattern matching

text  $T$



pattern  $P$

**Compute number of mismatching pairs**

**for each prime  $q$  in  $[\log m, \log^2 m]$  do**

**for each (substream, subpattern) do**

run streaming exact pattern matching



# From 1 mismatch to exact pattern matching

text  $T$



pattern  $P$

## Complexity

$$\text{Space} = O\left( \underbrace{\log m}_{\# \text{ of primes}} \cdot \underbrace{\log^2 m}_{\# \text{ of substr.}} \cdot \underbrace{\log^2 m}_{\# \text{ of subpatterns}} \cdot \log m \right)$$

$$\text{Time} = O\left( \underbrace{\log m}_{\# \text{ of primes}} \cdot \underbrace{\log^2 m}_{\# \text{ of substr.}} \cdot \underbrace{\log^2 m}_{\# \text{ of subpatterns}} \right)$$

# Approximate pattern matching (Hamming distance)

**Porat & Porat, 2009**

$\tilde{O}(k^3)$  space,  $\tilde{O}(k^2)$  time

Same as for  $k = 1$  but take more primes

**Clifford, Fontaine, Porat, Sach, S., 2016**

$\tilde{O}(k^2)$  space,  $\tilde{O}(\sqrt{k})$  time

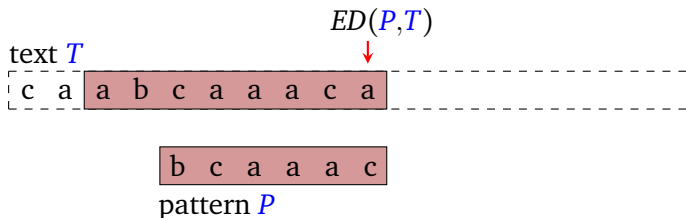
We can take fewer primes if we choose them at random + periodicity to improve time

**Clifford, Kociumaka, Porat, 2018**

$O(k \log \frac{m}{k})$  space,  $O(k \log^3 m \log \frac{m}{k})$  time

New encoding for mismatch information + periodicity + exponentially growing prefixes

# Approximate pattern matching (edit distance)

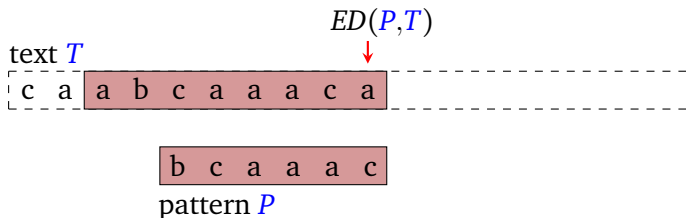


$ED(P, S)$  = minimum number of insertions, deletions, and replacements that transform  $P$  into  $S$

Example:  $P = aaac$ ,  $S = abacb$ , edit distance = 2

- ▶ If  $ED(P, T) > k$ , output “NO”
- ▶ Otherwise, output  $ED(P, T)$

# Approximate pattern matching (edit distance)



$ED(P, S)$  = minimum number of insertions, deletions, and replacements that transform  $P$  into  $S$

Example:  $P = \text{aaac}$ ,  $S = \text{abacb}$ , edit distance = 2

- ▶ Hybrid dynamic programming:  $\mathcal{O}(m)$  space,  $\mathcal{O}(k)$  time
- ▶ S., 2017:  $\mathcal{O}(\sqrt{m} \cdot \text{poly}(k, \log m))$  space,  $\mathcal{O}(\sqrt{m} \cdot \text{poly}(k, \log m))$  time

# Embedding from edit to Hamming distance

Chakraborty, Goldenberg, Koucky, 2016

Pick  $3n$  random functions  $h_j : \{0, 1\} \rightarrow \{0, 1\}$

	1	2	3	4	5	6	7	8	...	$3n$
0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

	1	2	3	...	$n$
$S$ :	0	1	0	...	0
$S'$ :				...	

**text position = 1, j = 1**

1. Copy  $S[i]$ . If  $h_j(S[i]) = 1$ , move to the right;
2.  $j = j + 1$ .

# Embedding from edit to Hamming distance

Chakraborty, Goldenberg, Koucky, 2016

Pick  $3n$  random functions  $h_j : \{0, 1\} \rightarrow \{0, 1\}$

	1	2	3	4	5	6	7	8			$3n$
0	0	1	1	0	1	1	0	0	...		0
1	1	1	1	1	0	1	0	1	...		1
									...		

Copy letters of  $S$  to  $S'$ :

	1	2	3			$n$
$S$ :	0	1	0	...		0
$S'$ :	0			...		

**text position = 1, j = 1**

1. Copy  $S[i]$ . If  $h_j(S[i]) = 1$ , move to the right;
2.  $j = j + 1$ .

# Embedding from edit to Hamming distance

Chakraborty, Goldenberg, Koucky, 2016

Pick  $3n$  random functions  $h_j : \{0, 1\} \rightarrow \{0, 1\}$

	1	2	3	4	5	6	7	8	...	$3n$
0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

$S$ :	0	1	0	...	0
$S'$ :	0			...	

**text position = 1, j = 1**

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0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

	1	2	3	...	$n$	
$S$ :	0	1	0	...	0	<b>text position = 1, j = 2</b>
$S'$ :	0					

1. Copy  $S[i]$ . If  $h_j(S[i]) = 1$ , move to the right;
2.  $j = j + 1$ .



# Embedding from edit to Hamming distance

Chakraborty, Goldenberg, Koucky, 2016

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0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

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Copy letters of  $S$  to  $S'$ :

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Pick  $3n$  random functions  $h_j : \{0, 1\} \rightarrow \{0, 1\}$

	1	2	3	4	5	6	7	8	...	$3n$
0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

	1	2	3	...	$n$	
$S$ :	0	<b>1</b>	0	...	0	
$S'$ :	0	0				<b>text position = 2, j = 3</b>

1. Copy  $S[i]$ . If  $h_j(S[i]) = 1$ , move to the right;
2.  $j = j + 1$ .

# Embedding from edit to Hamming distance

Chakraborty, Goldenberg, Koucky, 2016

Pick  $3n$  random functions  $h_j : \{0, 1\} \rightarrow \{0, 1\}$

	1	2	3	4	5	6	7	8	...	$3n$
0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

	1	2	3	...	$n$	
$S$ :	0	<b>1</b>	0	...	0	
$S'$ :	0	0	1	...		<b>text position = 2, j = 3</b>

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# Embedding from edit to Hamming distance

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0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

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$S$ :	0	1	0	...	0	
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0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

Copy letters of  $S$  to  $S'$ :

	1	2	3	...	$n$	
$S$ :	0	1	0	...	0	
$S'$ :	0	0	1	...		<b>text position = 2, j = 3</b>

If  $ED(S, T) = k$ , then  $k/2 \leq HD(S', T') \leq \mathcal{O}(k^2)$  w/ prob. 0.99

# Embedding from edit to Hamming distance

Chakraborty, Goldenberg, Koucky, 2016

Pick  $3n$  random functions  $h_j : \{0, 1\} \rightarrow \{0, 1\}$

	1	2	3	4	5	6	7	8	...	$3n$
0	0	1	1	0	1	1	0	0	...	0
1	1	1	1	1	0	1	0	1	...	1

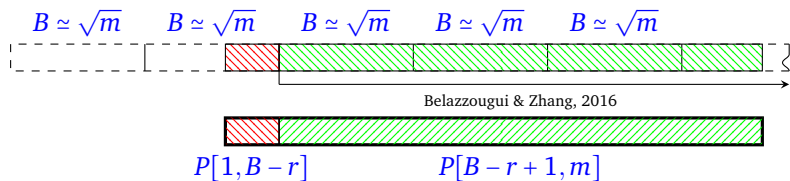
Copy letters of  $S$  to  $S'$ :

	1	2	3	...	$n$	
$S$ :	0	1	0	...	0	
$S'$ :	0	0	1	...		<b>text position = 2, j = 3</b>

Belazzougui, Zhang, 2016

- ▶ Embedding + streaming alg'm for  $k^2$ -mismatch  $\Rightarrow$  a good estimate for edit distance
- ▶ If  $ED(S, T) \leq k$ ,  $\tilde{O}(k^2)$  embeddings + streaming alg'm for  $k^2$ -mismatch  $\Rightarrow$  **exact value!**

# Approximate pattern matching (edit distance)



Starting from each block  $i$ , run Belazzougui & Zhang, 2016

$$ED[j] = \min_{i \in [r-k, r+k]} ED(P[1, B-i], \mathbf{T}_1) + ED(P[B-i+1, m], \mathbf{T}_2)$$

We compute  $ED(P[1, B-i], \mathbf{T}_1)$  while reading  $\mathbf{T}_1$  using dynamic programming, then encode the distances to restore later



## Part III: Preprocessing

# Preprocessing for pattern matching

Can we preprocess the patterns in a streaming way?  
If yes, do we need to read them several times?  
How much space do we need?

## Periodicity — Ergün, Jowhari, Saglam, 2010

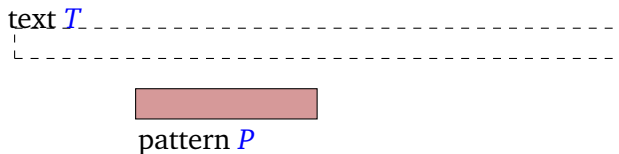
- ▶ Periodic patterns:  $O(\log m)$  space,  $O(\log m)$  time
- ▶ Non-periodic patterns:  $\Omega(m)$  space
- ▶ 2 passes (periodic and non-periodic patterns):  $O(\log m)$  space,  $O(\log m)$  time

## Periodicity with mismatches — Ergün et al., 2017

- ▶ Periodic patterns:  $O(k^4 \log^9 n)$  space
- ▶ 2-pass algorithm for non-periodic patterns, lower bounds

## Part IV: Property testing model

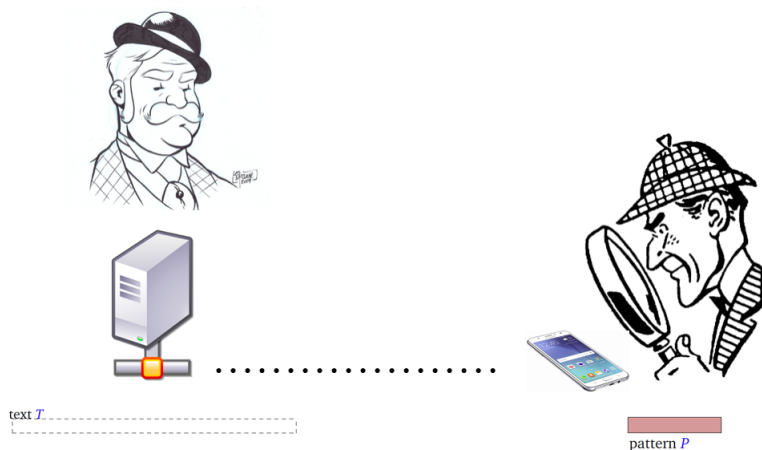
# Pattern matching



Is  $T$  **free** from occurrences of  $P$ ?

Same question when  $T$  and  $P$  are of dimension  $d \geq 2$

# Property testing model



If Sherlock wants to solve the problem fast, he can only query a few characters of  $T$

# Property testing model

**Task:** develop an ultra-efficient randomised algorithm to decide whether  $T$  is free from occurrences of  $P$

**We must**

- ▶ accept, if  $T$  is  $\varepsilon_1$ -close to being  $P$ -free
- ▶ reject, if  $T$  is  $\varepsilon_2$ -far from being  $P$ -free
- ▶ accept or reject otherwise

$\varepsilon_1$ -close = we can fix  $\leq \varepsilon_1 n$  characters of  $T$  so that the property is satisfied

$\varepsilon_2$ -far = we must fix  $\geq \varepsilon_2 n$  characters of  $T$  so that the property is satisfied

# Property testing model

**Task:** develop an ultra-efficient randomised algorithm to decide whether  $T$  is free from occurrences of  $P$

**We must**

- ▶ accept, if  $T$  is  $\varepsilon_1$ -close to being  $P$ -free
- ▶ reject, if  $T$  is  $\varepsilon_2$ -far from being  $P$ -free
- ▶ accept or reject otherwise

**Ben-Eliezer, Korman, Reichman, 2017**

There is an algorithm which queries  $O(\varepsilon^{-1})$  letters of  $T$  and distinguishes between  $\varepsilon/2$ -close and  $\varepsilon$ -far (for almost all patterns)

# Summary of today's talk

It's all about **pattern matching**

Randomisation and approximation  $\Rightarrow$  more efficient algorithms

Many open questions

Thank you!